

PROBLEM:

A 25- μC point charge is located at the origin. Calculate the electric flux passing through:

(a) that portion of the sphere $r = 20$ cm bounded by $\theta = 0$ and Π , $\phi = 0$ and $\Pi/2$; (b) the closed surface $\rho = 0.8$ m, $Z = \pm 0.5$ m.

SOLUTIONS :

(a) $r = 20$ cm = 0.2 m

The area of the strip is given by,

$$\begin{aligned} A &= \int_0^{\Pi} \int_0^{\frac{\Pi}{2}} r^2 \sin\theta \, d\theta \, d\phi \\ &= \int_0^{\Pi} \int_0^{\frac{\Pi}{2}} 0.2^2 \sin\theta \, d\theta \, d\phi \\ &= \int_0^{\Pi} -0.04 \left(\cos\theta \right) \Big|_0^{\frac{\Pi}{2}} d\phi \\ &= \int_0^{\Pi} -0.04 (-1 - 1) \, d\phi \\ &= \int_0^{\Pi} 0.08 \, d\phi = 0.08 \left(\phi \right) \Big|_0^{\Pi} = 0.08 \Pi = 0.2513 \text{ m}^2 \end{aligned}$$

Then the flux through the strip is,

$$\begin{aligned} \Psi_{\text{net}} &= \frac{A}{4\pi r^2} Q \\ &= \frac{0.2513}{4\pi(0.2)^2} \frac{25 \times 10^{-6}}{0.2^2} \\ &= 6.25 \times 10^{-6} \text{ C or } 6.25 \mu\text{C} \end{aligned}$$

(b)

$$\Psi = \int_{-0.5}^{0.5} \int_0^{2\pi} \rho \, d\theta \, dZ$$

but $\int_{-0.5}^{0.5} \int_0^{2\pi} \rho \, d\theta \, dZ = \frac{Q}{2\pi\rho}$

$$= \frac{Q}{2\pi\rho} \int_{-0.5}^{0.5} \int_0^{2\pi} \rho \, d\phi \, dZ$$

$$= \frac{25 \times 10^{-6}}{2\pi} \cdot 2\pi \int_{-0.5}^{0.5} dZ$$

$$= 25 \times 10^{-6} (0.5 + 0.5)$$

$$= 25 \times 10^{-6} \text{ C or } 25 \mu\text{C}$$